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The v.l.f. admittance of a dipole in the lower ionosphere

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Abstract. The admittance of a dipole in the lower ionosphere has been calculated for very low frequencies. Both the real and reactive parts of the admittance are found to be related to the ion sheath which forms about the dipole and are functions of electron density and temperature of the ionosphere. An electron density profile is estimated based upon measurements reported in an earlier rocket flight.

AUTHOR

1. Introduction

At radio frequencies exceeding the local plasma frequency it has been demonstrated that the impedance (or admittance) of a dipole in the ionosphere is affected by the ion sheath which forms around it (Jackson and Kane 1959). The measured impedance is useful not only in determining the radiation properties of the dipole, but also in determining the local electron density. Another device used to measure directly the ion density and electron temperature in the ionosphere is the dumbbell probe (Spencer, Brace and Carignan 1962). This instrument operates as a symmetrical Langmuir probe in which the measurements are determined from the d.c. characteristics of the device. In this paper, we shall attempt to calculate the admittance, at very low frequencies, of a dipole immersed in the ionosphere. At frequencies below the plasma frequency, the dipole may be viewed as a symmetrical Langmuir probe. In this case the terminal admittance appears to be determined by the behaviour of the ion sheath which forms about the dipole. Again, the admittance will be related to the local electron density and temperature.

Measurements of the complex admittance of a dipole in the ionosphere have been recently reported by Lomax (1961). A Nike–Cajun rocket was fired nearly vertically at about 1700 LT, 14th March, 1961, from the Eglin Gulf Test Range, Florida. On the second stage Cajun rocket the payload and motor casing were separated by a dielectric sleeve and the entire second-stage rocket body was used as an approximately symmetrical electric dipole. The admittance across the dielectric gap was measured up to an altitude of about 100 km and again on the downward trajectory. The calculations of admittance in this paper will relate to this particular dipole and rocket flight, although, in most cases, the concepts and equations are of more general applicability.

2. Analysis

2.1. *Origin of terminal admittance*

Any body immersed in a neutral plasma rapidly acquires a negative charge because the random electron currents are greater than the random ion currents. The accumulation of charge is limited by the negative potential which develops between the body and the medium, and equilibrium with the medium is established when the electron and ion currents are the same. This situation is illustrated in figure 1(*a*) in which the electron and ion currents are shown to be in equilibrium for each rocket half; the same potential is established

with the radio frequency period, in which case the electrons may have little influence on the dipole admittance and the heavy positive ions (neglected in the present analysis) may be of primary importance.

Reference

RATCLIFFE, J. A., 1959, *The Magneto-ionic Theory and its Applications to the Ionosphere* (Cambridge: University Press).

between each rocket half and the ionosphere; and an identical ion sheath develops about each element. No current flows between the rocket halves.

When a small d.c. potential δV is applied across the dielectric gap, half of the potential appears across each sheath as shown in figure 1(*b*). The upper element is made more positive and the magnitude of the electron current is increased by *dI*, while the opposite changes are made at the lower element. The random ion currents are unchanged by the small d.c. potential. Also, the sheath radii are decreased or increased by *dR*, as shown.

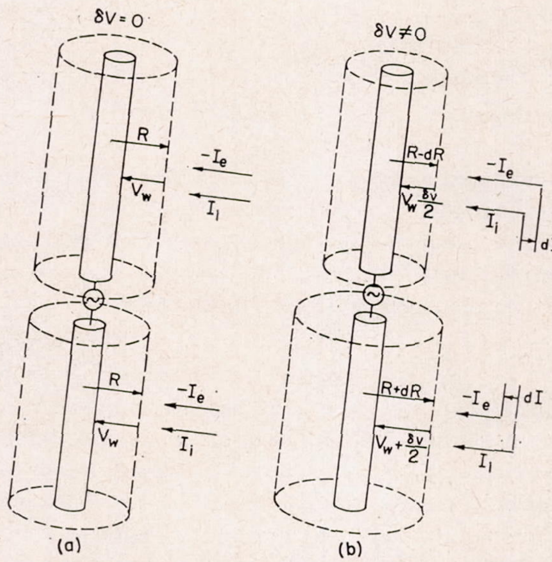

Figure 1. Showing the ion sheath which forms about a dipole in the ionosphere with (*a*) zero potential between the elements, (*b*) a small potential between the elements.

At the very low frequencies considered here (16·5 kc/s in the rocket experiment) it is believed that the ion sheaths will have ample time to respond fully to the applied voltage variations. This quasi-static condition exists for the lowest altitudes because the relaxation time of the electrons for this collision-dominated plasma is less than the period of the applied voltage (Delcroix 1960). At higher altitudes, even though the collision frequency may be small, the plasma frequency is greater than the applied frequency and quasi-static conditions should still pertain (Rose and Clark 1961). Thus, in response to a small oscillating δV, a component of current *in phase* with the applied voltage will flow. This current represents a conductance, given by

$$G = \frac{\delta I}{\delta V} = \frac{1}{2}\frac{dI}{dV} \tag{1}$$

where δI and δV are measured at the generator, and dI and dV are measured across the sheath. The factor $\frac{1}{2}$ is necessary because half the applied voltage δV appears across each sheath for small signals. The derivatives must, of course, be evaluated at the equilibrium potential V_w. It may be seen that this conductance corresponds to the slope of the static characteristics of the dipole when treated as a symmetrical Langmuir probe.

The applied potential also results in variations of the sheath dimensions. When the sheath radius R varies, the number of ions in the sheath changes. This incremental change in ion charge must be neutralized by an equal change in electron charge on the dipole elements. Thus, another component of current is required to place the appropriate charge on each dipole element which is consistent with the variations of sheath dimension. This current is in *phase quadrature* with the applied voltage and accounts for the reactive part (capacitative) of the admittance. This capacitance may be found from the relation for incremental capacitance,

$$C = \frac{\delta Q}{\delta V} = \frac{1}{2}\frac{dQ}{dV}. \tag{2}$$

The capacitance is analogous to the depletion layer capacitance observed across a semiconductor junction.

For dipoles short compared to a wavelength, the radiation resistance is presumed to be negligible and the terminal admittance measured will consist solely of the parallel conductance and capacitance given in equations (1) and (2) respectively. The effect of the dipole motion, the earth's magnetic field, and shock waves will be included below by estimating their effects on the currents flowing to the dipole.

2.2. *Current flow*

It will be assumed throughout this paper that, because of sufficient collisions, the neutrals, ions and electrons are in thermal equilibrium in the ambient medium. Initially, the earth's magnetic field will be neglected and the charged particle motions will be assumed to be isotropic. Ion and electron current expressions will be developed which will be applicable to the ionosphere. To simplify the discussion, the D region will imply the altitude range 65 km to 80 km, and the E region will imply altitudes above 80 km.

Because the electron thermal velocities are much greater than the rocket velocity, the motion of the rocket has a negligible effect on electron currents. For the quasi-static situation in the E region, where the mean free path is larger than the rocket and sheath dimension, the electron current to a surface at potential V with respect to the plasma has been calculated to be (Hoegy and Brace 1961)

$$I_e = ANq\left(\frac{kT}{2\,\pi m_e}\right)^{1/2} \exp\,(qV/kT) \text{ amps} \tag{3}$$

where A is the element area, N the electron density, m_e the electron mass, q the electron charge, T the electron temperature, and k is Boltzmann's constant (m.k.s. rationalized units will be used throughout). For each dipole half $A = 2\,\pi R_0 \times L/2$, where R_0 is the rocket radius and L is the rocket length. For the D region the mean free path is small compared to the sheath dimension. Assuming for this case that potential energy is conserved for electron–neutral collisions within the sheath, and that the electrons are in thermodynamic and potential equilibrium within the potential well of the sheath, the quasi-static electron current at the rocket surface is again given by equation (3).

For the ion current to the dipole, the rocket motion through the ionosphere is significant since it is at least comparable with the ion thermal velocities. The ion density is not able to adjust to potential equilibrium within the sheath in one period of the applied voltage, and can be considered to be at ambient density within the sheath. If the rocket were moving

fast enough, the ions would be merely swept up. However, for the rocket flight of interest here, the velocity was not that large and the ion flux perpendicular to the direction of rocket motion was significant. An equation for the random current density to a moving infinitesimal area element was given by Hoegy and Brace (1961). Their expression has been applied to the case of a rocket with a square cross section moving broadside through the ionosphere. The resulting approximation is

$$I_{\mathrm{i}} = \tfrac{1}{4}\, ANq \left[W + \left(\frac{2\,kT}{\pi m_{\mathrm{i}}} \right)^{1/2} \right] \tag{4}$$

where W is the rocket velocity, m_{i} is the ion mass and the remaining symbols are as given for equation (3). The rocket was assumed to be oriented perpendicularly to its velocity direction, and again $A = 2\,\pi R_0 \times L/2$. This expression is applicable to both the D and E regions.

The effects of the earth's magnetic field on this problem are twofold. (i) The field influences the motions of the charged particles and hence the random currents. (ii) A voltage is induced on the dipole because of its motion through this field (Beard and Johnson 1960, Bourdeau *et al.* 1961). The first effect may be summarized as follows: (*a*) The isotropy of the ion motions to the dipole is unaffected by the field because the ion collision frequency is greater than the ion gyro frequency and the ion gyro radius is greater than the rocket and sheath diameter. (*b*) The isotropy of the electron motions in the D region are unaffected since the electron collision frequency is greater than the electron gyro frequency. However, this is not true for the E region. In the E region the mean electron motion may be assumed to be along the magnetic field since the electron gyro radius is smaller than the rocket and sheath diameter. The second effect of the magnetic field will be neglected here because the upward rocket trajectory was nearly parallel to the magnetic field and hence the induced electric field was small. Furthermore, the downward trajectory showed no significant differences from the upward trajectory.

In the D region, where the mean free paths are small, the gas motions are hydrodynamic and, since the rocket velocity is supersonic, shock waves can be expected. To consider shock waves to a first order, it was assumed that the sheath was within the detachment distance, and that the currents flowing to the forward side of the rocket were from a hotter, more dense ionosphere moving more slowly with respect to the rocket. Temperatures, densities and subsonic velocities within the detachment distance were calculated for the normal shock wave from well-known relations (Liepman and Roshko 1957) and are believed to represent the case which would have the greatest effect on current flow.

The expressions showing the modification to the electron current in the E region because of the earth's magnetic field, and the shock wave modifications to the ion and electron currents in the D region will not be given here. However, these effects on the calculations of G and C will be included and discussed.

2.3. *Equilibrium potential and sheath radius*

Since equations (1) and (2) must be evaluated at the equilibrium potential and sheath radius, respectively, it will be necessary to obtain expressions for these parameters. The equilibrium potential V_{w} between the dipole and the ionosphere is obtained by equating

equations (3) and (4) and is given by

$$\exp\left(qV_w/kT\right) = \frac{\left[W + \left(\frac{2\,kT}{\pi m_i}\right)^{1\,2}\right]}{4\left(\frac{kT}{2\,\pi m_e}\right)}. \tag{5}$$

The expression for sheath radius is somewhat more elusive, as is the definition of sheath radius itself. In principle, Poisson's equation may be employed to obtain the potential distribution about a body immersed in a plasma. Some useful numerical solutions of Poisson's equation for a moving sphere have been obtained (Davis and Harris 1961). Jastrow and Pearse (1957) have obtained a solution to Poisson's equation in spherical coordinates by assuming that the sheath consists of a sharp boundary within which there are no electrons and that the ion density is constant at the ambient value. A solution of Poisson's equation in cylindrical coordinates with the same sheath model is

$$2\,\epsilon_0 V = -\,NqR^2 \ln\left(R/R_0\right) + (Nq/2)(R^2 - R_0{}^2). \tag{6}$$

2.4. *Predicted admittance*

The expression for the conductance which is applicable to the rocket experiment can be obtained from equations (1), (3) and (4). Carrying out the indicated operations yields

$$G = \frac{1}{2}\frac{d}{dV}(I_e - I_i) = \frac{q}{2\,kT}I_e. \tag{7}$$

The total charge in the sheath about each dipole element is

$$Q = -\,Nq\,\pi(R^2 - R_0{}^2)\,L/2. \tag{8}$$

The expression for capacitance is then obtained by differentiating equations (6) and (8) with respect to R and substituting in the following equation.

$$C = \frac{1}{2}\frac{(dQ/dR)}{(dV/dR)} \tag{9}$$

which yields

$$C = \frac{\pi\epsilon_0 L}{2\ln\left(R/R_0\right)}. \tag{10}$$

The parallel combination of G and C represents the desired terminal admittance of the rocket dipole in the lower ionosphere.

Equations (7) and (10) neglect the earth's magnetic field and shock waves. Within this approximation, the expected admittance for the rocket-body dipole was calculated for an assumed model of the lower ionosphere. In this model, N increased exponentially from 10^8 electrons/m^3 at 70 km to 10^{10} electrons/m^3 at 90 km, the temperature was constant at 200°K above 80 km and gradually increased to 250°K as the altitude decreased to 65 km. The ionic mass was assumed to be 30 (corresponding to NO^+). The results are shown in figure 2 as plots of G and C against altitude.

The effects of the magnetic field on the electron motions in the E region and shock wave effects in the D region are also shown in the calculations of figure 2 by the primed and

double-primed curves, respectively. It can be seen that the magnetic field has a negligible effect and that shock waves make less than a factor of 2 difference in G and a negligible difference in C. Thus, the simplified calculations will be used for a first order comparison with the measurements of the rocket experiment.

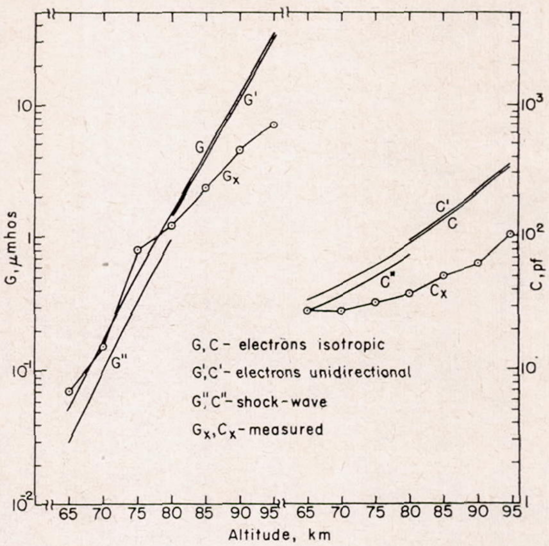

Figure 2. Calculated values of the admittance parameters with electron motion assumed isotropic (G, C), with the mean electron motion assumed to be parallel to the magnetic field above 80 km (G', C') and including the effect of a shock wave below 80 km (G'', C''). The smoothed measured values of Lomax (1961) are shown as G_X and C_X.

2.5. *Comparison of predictions and measurements*

From the measurements reported by Lomax (1961), smoothed values were read at 5 km height intervals by averaging over the modulation produced by the rocket motion. These data are shown in figure 2 and are labelled G_x and C_x. The comparison between G_x and G and between C_x and C is not at all bad considering that no undetermined constant factors of any kind have been employed in the analysis presented above.

In order to ascertain the source of the relatively small differences shown in figure 2, it would be useful to obtain simplified expressions for G and C in terms of N and T. Approximations to equations (7) and (10) are possible for the case of small sheaths and for assumptions regarding the rocket velocity. The approximations leave much to be desired in showing a simple dependence of G and C on temperature T. However, it may be shown that

$$G \propto N; \quad C \propto N^{1/2} \tag{11}$$

when the sheath is small.

On the assumption that T is more closely known than is N in the lower ionosphere, the electron density profile was adjusted using equation (11) to minimize the difference between G and G_x and between C and C_x. The revised profile is labelled N_r in figure 3(a) and may be compared with the initial model labelled N. The calculated values for G_r and C_r which correspond to the profile N_r are shown in figure 3(b) and may be com-

pared with the experimental values. The remarkable agreement shown here lends support to the previous analysis. The electron density profile in figure 3(*a*) is quite comparable with those deduced by other workers, as conveniently summarized by Titheridge (1962).

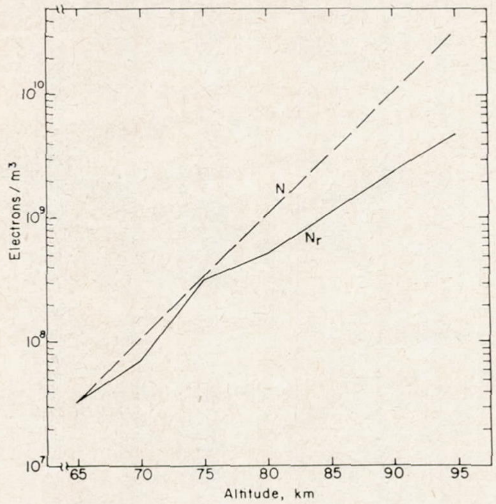

Figure 3(*a*). The initial model N and the revised N_r electron density profiles.

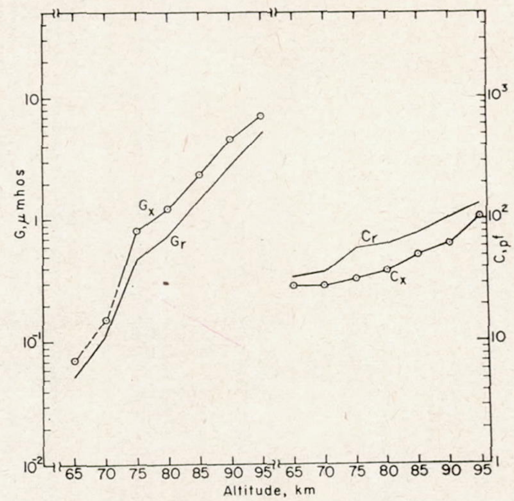

Figure 3(*b*). Calculated values of the admittance parameters corresponding to N_r (G_r and C_r) and the smoothed measured values (G_x and C_x).

3. **Discussion and conclusions**

There are several additional factors which a more thorough analysis should include. First, the collecting area for the positive ions is more nearly determined by the sheath radius rather than by the rocket radius. Second, in the lowest part of the ionosphere the sheath radius will become quite large because of low electron densities. In this case, the capacitance between the dipole elements and within the sheath becomes important. In fact, this capacitance eventually dominates the capacitance resulting from the sheath charge variation and approaches the free-space value as the electron density approaches zero. These effects have been neglected in this preliminary analysis in order to simplify the work and it is believed that they do not seriously alter any of the conclusions.

In summary, it has been possible to calculate, to a first order, the terminal admittance of a split-rocket dipole moving through the ionosphere. The dipole was viewed as a symmetrical Langmuir probe and the expressions for admittance so obtained are a function of the dipole dimensions, the dipole velocity, and the ionic composition, electron density and temperature of the ionosphere. Calculations based upon these expressions were compared with experimental values obtained on a recent rocket flight. The agreement is remarkably good and the measured values of admittance imply an electron density profile consistent with that deduced by various other means. Electron densities in the D region of less than $10^8\,\mathrm{m}^{-3}$ may be measured in this way. A v.l.f. admittance probe is believed to be a new and useful device for the measurement of the electron density and possibly the temperature of the ionosphere.

Acknowledgments

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